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## Dealing with stiffness: shaft dynamics in the golf swing

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### Abstract

A study was undertaken to investigate the influence of shaft stiffness on the deflection and the positioning of the club face during a golf swing. To accomplish this task, a theoretical model was developed that calculated deflections of golf shafts during the swing. Applying moment-curvature in relation to a shaft that had the bending stiffness, a differential equation and the boundary conditions that characterized its behaviour over the duration of the swing were generated. Incorporated into the bending moment-distribution are forces and torques acting at each end of the shaft. These loads were calculated using an inverse dynamics problem of the golf club during the swing. A motion capture system was utilized to collect the necessary data. Solutions to the equations of motion for the shaft were calculated numerically. The results have been presented as deflections along the shaft as functions of time. Bending deflections were determined in the plane of the club face and normal to this plane were calculated.

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### 1. Introduction

Stiffness distribution of golf club plays an important role in the design and the appropriate choice of shafts. The more flexible shaft is conventionally suited to the weaker player, while the professional or good amateur should use a stiffer one (Milne and Davis (1992)). Computer simulations predict the bending of shaft during swing and provide the player with optimal shaft. However, the guideline for choosing appropriate stiffness of the shaft is complex since the physical meaning of shaft stiffness in golf swing is still uncertain. This study was executed to get comprehensive understanding of the dynamic behavior of a golf shaft during the swing. The aim is to develop a

mathematical model that describes the deflections occurring in the golf shaft based on static deflection model due to three-dimensional force and torque that is calculated from motion capture data.

## 2. Eccentric payload model

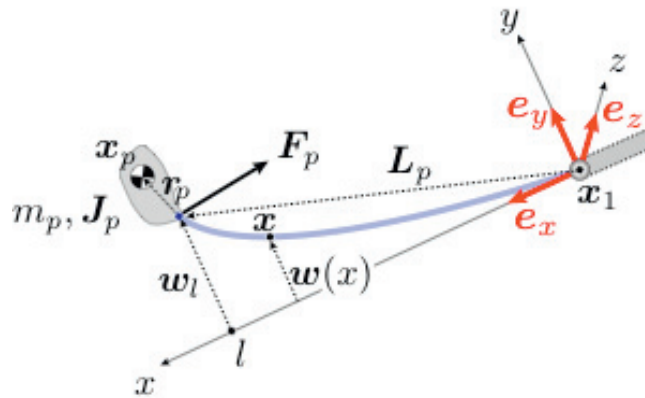


Fig. 1. Eccentric payload model of golf club.

Fig. 1 shows an eccentric payload model of a golf club and a frame fixed to the shaft with its origin located at the grip. The deflections are defined with respect to the frame with the  $x$  axis along the neutral axis of the shaft, the  $y$  axis perpendicular to the shaft and is in the plane of club face, and the  $z$  axis completing the right-hand orthogonal system. Torsional deformation is measured about the  $x$  axis. The total bending deflection of the shaft is described in terms of  $y$  and  $z$  components. The deflection along the  $y$  axis represents toe-up/toe-down motion, while deflection along the  $z$  axis represents lead/lag motion at impact. The shaft is deflected in the both directions during the downswing (MacKenzie and Sprigings (2009)).

The club shaft as mathematical model is often regarded as a cantilever beam. If the shaft is cantilever beam, both deflections are functions of time and position along the shaft and the equation of motion is described by a fourth order partial differential equation (Brylawski (1994), McGinnis and Nesbit (2010)). In order to solve the partial differential equation, a numerical method must be employed in order to find the deformation of shaft.

During the downswing radial centripetal force acting on the center of mass of the club head and the centripetal force acts greater as it approaches the impact. This radial force that acts on the club head causes the forward/toe-down bending moment because of eccentric payload, which has the offset position of the center of mass of the club head relative to the central axis of the shaft. If the moment acts strongly at the tip end of the shaft, the shaft model must be described as a both ends fixed beam and the bending vibration can be neglected relative to the deflection caused by the moment acting at both the tip and the butt ends of the shaft.

The position vector of the origin of the frame  $\mathbf{x}_1(t)$  has 3 degrees of freedom and this moving frame also has 3 rotational degrees of freedom. The club shaft which is fixed to the club head at the tip end has stiffness distribution  $EI(x)$ . The position vector of the club head  $\mathbf{x}_p(t)$ , deflection displacement vector at the tip end  $\mathbf{w}_l(t)$ , and the position vector of the center of the mass of the club from the origin of the frame  $\mathbf{L}_p$  are defined as follows,

$$\begin{aligned}\mathbf{x}_p(t) &= \mathbf{x}_1(t) + l\mathbf{e}_x(t) + \mathbf{w}_l(t) + \mathbf{r}_p(t), \\ &= \mathbf{x}_1(t) + \mathbf{L}_p(t)\end{aligned}\tag{1}$$

$$\mathbf{w}_l(t) = y(t)\mathbf{e}_y(t) + z(t)\mathbf{e}_z(t)\tag{2}$$

$$\mathbf{L}_p(t) \equiv l\mathbf{e}_x(t) + \mathbf{w}_l(t) + \mathbf{r}_p(t),\tag{3}$$

where  $l$  is length of the shaft,  $\mathbf{r}_p(t)$  is the position vector from the tip end to the center of the mass of the club,  $\mathbf{e}_x(t)$ ,  $\mathbf{e}_y(t)$ ,  $\mathbf{e}_z(t)$  are the unit vectors in each axis of the moving frame. The force acting at the club head is expressed as

$$\begin{aligned}\mathbf{F}_p(t) &= m_p (\ddot{\mathbf{x}}_p(t) - \mathbf{g}) \\ &= m_p (\ddot{\mathbf{x}}_1(t) + \dot{\boldsymbol{\omega}}(t) \times \mathbf{L}_p + \boldsymbol{\omega}(t) \times (\boldsymbol{\omega}(t) \times \mathbf{L}_p) + \ddot{\mathbf{w}}_l(t) - \mathbf{g})\end{aligned}\quad (4)$$

where  $m_p$  is mass,  $\boldsymbol{\omega}$  is angular velocity of the club, and  $\mathbf{g}$  is the gravitational acceleration. Assuming that oscillation term  $\ddot{\mathbf{w}}_l$  is far less than other accelerations, we obtain

$$\mathbf{F}_p = m_p (\ddot{\mathbf{x}}_1 + \dot{\boldsymbol{\omega}} \times \mathbf{L}_p + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{L}_p) - \mathbf{g}) \quad (5)$$

The Euler equation of the club head is

$$\mathbf{J}_p \dot{\boldsymbol{\omega}} + \boldsymbol{\omega} \times (\mathbf{J}_p \boldsymbol{\omega}) = \mathbf{M}(l) - \mathbf{r}_p \times \mathbf{F}_p, \quad (6)$$

where  $\mathbf{J}_p$  is inertia tensor of the club head and  $\mathbf{M}(l)$  is the moment acting at tip end. Since dynamic term (left side) is considerably small than external force terms (right side) in Eq. (6), we get the concentrated moment at tip end

$$\mathbf{M}(l) = \mathbf{r}_p \times \mathbf{F}_p, \quad (7)$$

and similarly the concentrated moment at butt end

$$\mathbf{M}(0) = -\mathbf{L}_p \times \mathbf{F}_p. \quad (8)$$

Using the both concentrated moment, we finally obtain bending moment distribution in  $y$  and  $z$  directions  $\mathbf{M}_{yz}(x)$

$$\mathbf{M}_{yz}(x) = \begin{bmatrix} M_y(x) \\ M_z(x) \end{bmatrix} = \mathbf{M}(x)^T \begin{bmatrix} \mathbf{e}_z(x) \\ -\mathbf{e}_y(x) \end{bmatrix}, \quad (9)$$

and a moment curvature relationship

$$\begin{aligned}EI(x) \frac{d^2 \mathbf{w}(x)}{dx^2} &= \mathbf{M}_{yz}(x) \\ &= \frac{l-x}{l} (\mathbf{r}_p \times \mathbf{F}_p)^T \begin{bmatrix} \mathbf{e}_z(x) \\ -\mathbf{e}_y(x) \end{bmatrix} + \frac{x}{l} (-\mathbf{L}_p \times \mathbf{F}_p)^T \begin{bmatrix} \mathbf{e}_z(x) \\ -\mathbf{e}_y(x) \end{bmatrix} \\ &\equiv \frac{l-x}{l} \mathbf{M}_{yz}(0) + \frac{x}{l} \mathbf{M}_{yz}(l).\end{aligned}\quad (10)$$

### 3. Experiments

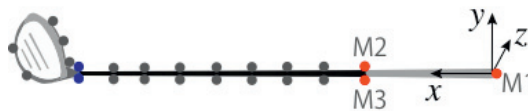


Fig. 2. Marker sets for measuring club shaft deflection and inverse dynamics problem.

Two male golfers participated in the present study. Each golfer tested driver and iron clubs. A motion capture system (Vicon MX series, UK) with 12 cameras is used to collect data at 500 Hz from the golfers' swings. The system tracks passive-reflective markers that are placed on the club. Golf clubs are equipped with several markers attached at different locations, as illustrated in Fig. 2. These position vectors are used to define the unit vectors and to measure deflection in  $y$  and  $z$  directions. Marker M1 and midpoint between markers M2 and M3, illustrated in Fig. 2, define the unit vector in the  $x$  direction which is parallel to the central axis of the shaft.

To determine the bending stiffness distribution  $EI(x)$  of the shaft, the load to the shaft, which is supported at two different points, associated with forced displacement was measured. The bending moment distribution  $M(x)$  as a function of  $x$  was determined by the load to the shaft. The curvature-moment relation in  $y$  direction

$$EI(x) \frac{d^2 y(x)}{dx^2} = M(x) \quad (11)$$

gives the deflection curve  $y(x)$  as a function of  $x$  and  $EI$  with given boundary conditions. Using this deflection curve, we finally obtain the bending stiffness distribution  $EI(x)$  at each point. Fig. 3 shows measured the distribution and a fitted curve. The fitted curve is composed by a linear and second order curve, which is smoothly connected and is extrapolated to the end of both tips. In the following simulations to predict the deflection of the shaft, we use this fitted curve.

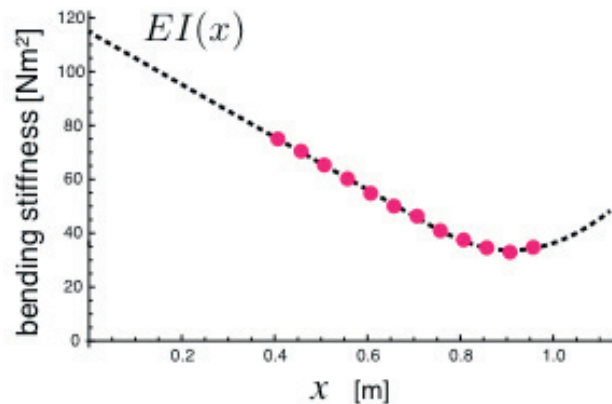


Fig. 3. Estimated bending stiffness distribution  $EI(x)$  of driver where  $x$  is displacement from grip end.

#### 4. Results

The swing data for the two subjects was analyzed. Time is from the initiation of the downswing until impact ( $t = 0$ ). The deflection  $w(t)$  during golf swing was predicted by Eq. (8) giving boundary conditions  $w(0.1)=w'(0.1)=0$  and substituting  $F_p$  which is calculated by Eq. (2). Fig. 4 shows measured deflection curve in typical driver swing and predicted one in  $y$  direction. Time is from the initiation of the downswing until impact ( $t = 0$ ). The scale of the  $y$  axis was magnified to 3 times the  $x$  axis. Fig. 6 shows the predicted and measured deflections at the tip end of the shaft. The standard deviations of the distance between the predicted and measured position at the tip end are 9.3 mm in  $y$  and 11.7 mm in  $z$  direction.

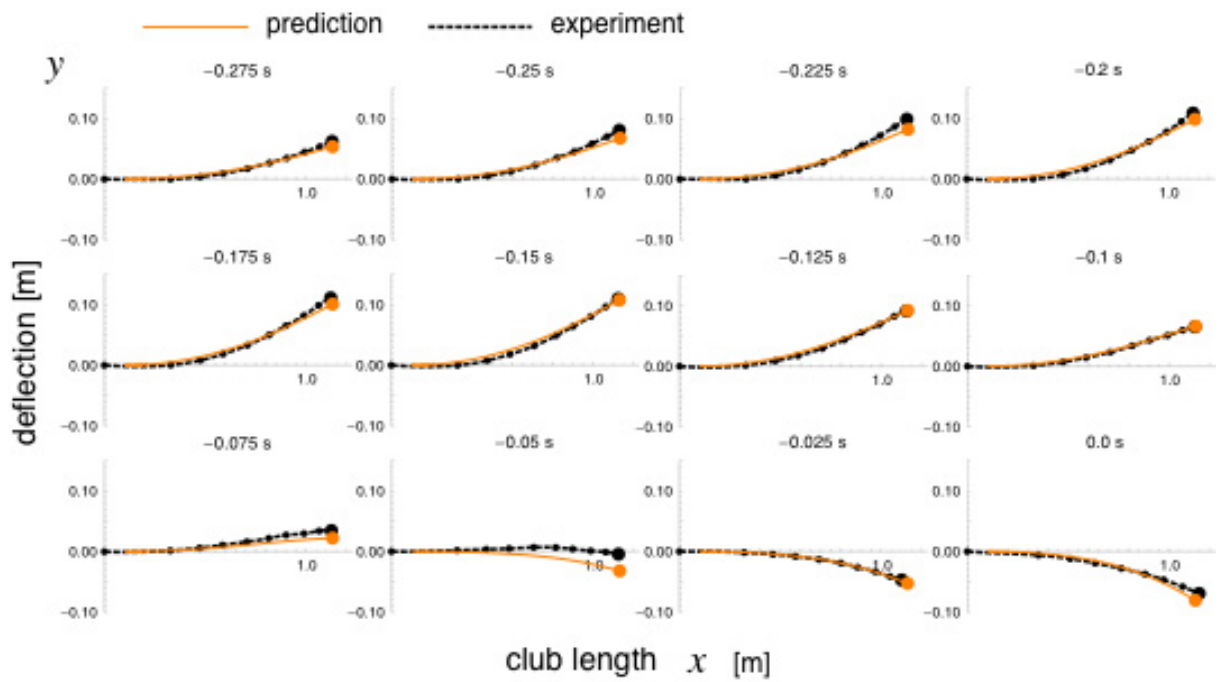


Fig. 4. Shaft deflections in  $y$  direction. The shaft deflection  $y$  magnified 3 times  $x$  axis.

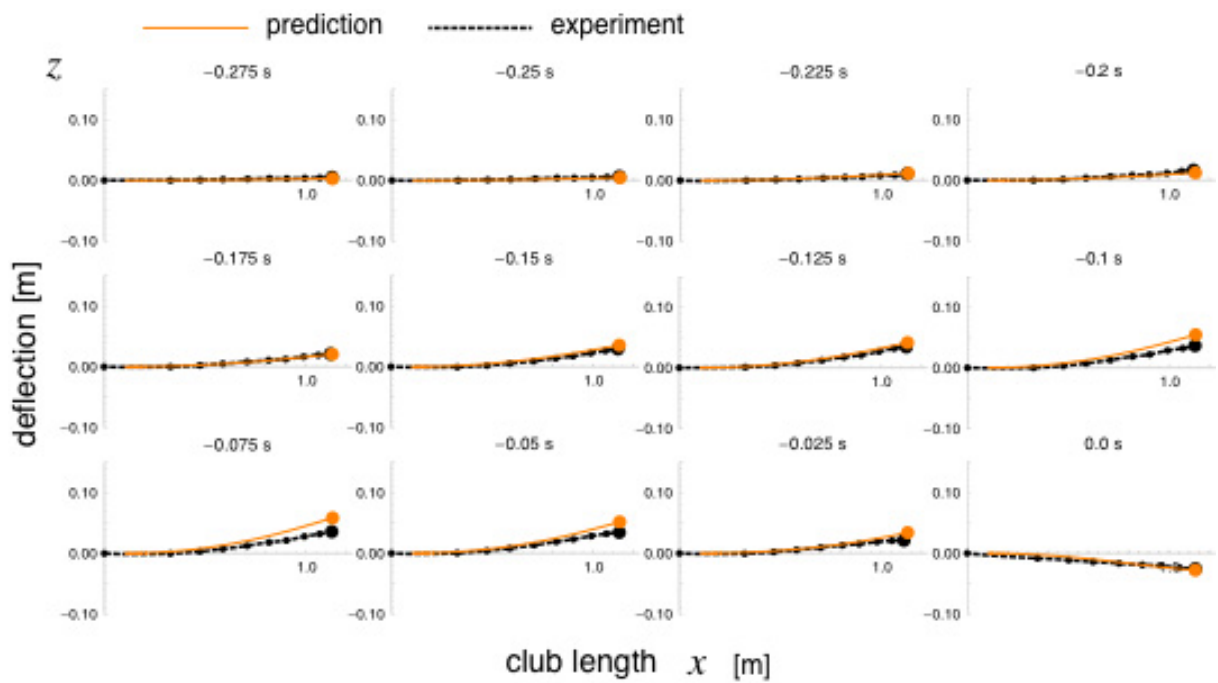


Fig. 5. Shaft deflections in  $z$  direction. The shaft deflection  $z$  magnified 3 times  $x$  axis.

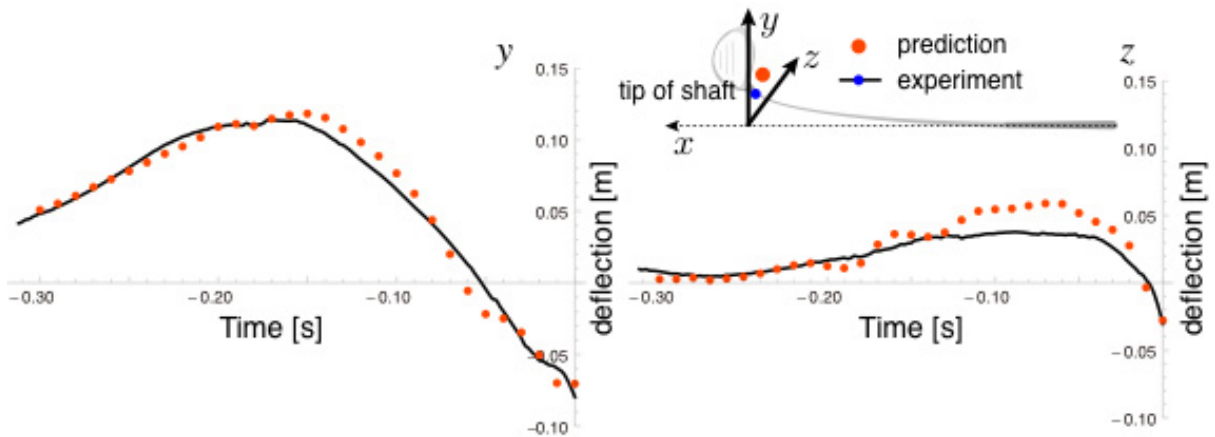


Fig. 6. Comparison of deflections at tip end of shaft.

#### 4. Discussion

The swing data for the two subjects was analyzed. The eccentric payload model predicts well the deflections of the driver shaft during the downswing. The oscillation of the shaft deflection can be neglected during the downswing, although the deflection progression looks similar as oscillating deflection which the cantilever beam model predicts. These results suggested that the shaft behave as not the cantilever beam but the both fixed one due to the strong moments at the both end of the shaft. The model predicts well the deflection of the iron club during the swing and the torsional deformation angle too.

This eccentric payload model gives us the benefit of not only the reduction of computation load but also the comprehension of the physical meaning of the deflection and the stiffness in the shaft. These understand also benefit appropriate choice of shaft in relation to the golfer.

#### References

- Milne, RD., Davis, JP., 1992. The Role of the Shaft in the Golf Swing., *Journal of Biomechanics* 25, pp. 975-983.
- MacKenzie, SJ., Spriggs, EJ., 2009. Understanding the role of shaft stiffness in the golf swing. *Sports Engineering* 12, pp.13-19.
- MacKenzie, SJ., Spriggs, EJ., 2010. Understanding the mechanisms of shaft deflection in the golf swing. *Sports Engineering* 12, pp.69-75.
- Brylawski, AM., 1994, An investigation of three dimensional deformation of a golf club during downswing, *Proc. of the World Scientific Congress of Golf*, pp. 265- 270.
- McGinnis, RS., Nesbit, S., 2010, Golf Club Deflection Characteristics as a Function of the Swing Hub Path, *The Open Sports Sciences Journal* 3, pp.155-164.